# EXPERIMENTAL AVAILABILITY TABLES FOR FINITE SPARES BACKLOGS

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# THESIS

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FOR FINITE SPARES BACKLOGS

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# Experimental Availability Tables
# for Finite Spares Backlogs

by

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Experimental tables of availabilities at time t are obtained for a device whose performance is described by an alternating renewal process with a finite number of failure-renewal cycles, corresponding to having a finite spares backlog. Failure and repair rates are assumed to be constant, and attention is restricted to cases in which the repair rate is larger than the failure rate.

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# I. INTRODUCTION

The most commonly encountered working definition of the "availability" of a device,

$$\text{(1.1)} \qquad \text{Availability} = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

represents the long-term or steady-state probability that the device will be found in an "up" or functioning condition when two specific conditions are satisfied. One condition is that there is an alternation of failure and repair cycles in which times to failure and times to repair are independent realizations from some failure and repair distributions satisfying minimal regularity conditions. The second, and here most important condition, is that the alternation of failure and repair continues indefinitely, so that the performance of the device is described by a standard alternating renewal process.

For many equipments, the second condition cited above implies access to an infinite backlog of spares. In many operational contexts this sort of spares support cannot be realistically assumed.

If availability is considered to derive from an alternating renewal process with a finite number of cycles, corresponding to a finite backlog of spares, then expressions for availability become complex as compared with equation (1.1).

This thesis is devoted to computational experiments with some "finite spares" availability expressions. The end objective of such experiments is to be able to determine the circumstances in which equation (1.1) furnishes an adequate approximation, or alternately to be able to provide computationally feasible alternatives to its use.

## II. MATHEMATICAL MODEL

The mathematical model on which the usual expressions for the availability of a device are based is an alternating renewal process with an infinite number of failure-repair cycles. In situations where repair requires replacement of a failured device by a spare, this corresponds to having an infinite number of spares. The model studied here is modified to allow only a finite number of failure-repair cycles, corresponding to having a finite number of spares.

The simplest assumptions about failure and repair times are made; failure rates are constant, and repair rates are constant. Only those processes that begin with a functioning device installed are considered.

In greater detail, the failure-repair process considered is as shown in figure 2.1,

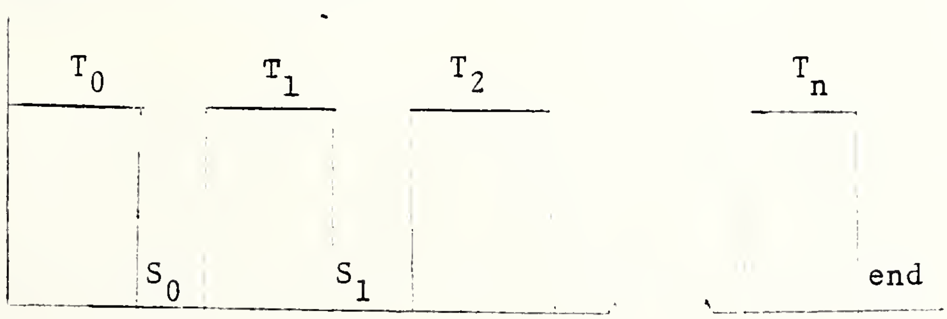


Figure 2.1 Failure-repair process.

where n is the number of spares, $T_0$ is the time to failure for the original device, $T_1$, $T_2$,......$T_n$ are the times to failure for the n spares, and $S_0$, $S_1$,......$S_{n-1}$ are the times to replace the original device and the first n-1 spares. It is assumed that $T_0$, $S_0$, ......$T_{n-1}$, $S_{n-1}$, $T_n$ are independent random variables, and that $T_0$,......, $T_n$ are exponentially distributed with failure rate $\lambda$, while $S_0$,......$S_{n-1}$ are exponentially distributed with repair rate $\eta$.

The availability at time t, $A_n(t)$, of the original device, supported by its backlog of n spares, is the probability that the process shown in Figure 2.1 is in an "up" condition at time t; i.e., that at time t either the original device or one of its spares is installed and still functioning.

The increment in availability at time t due to the $k^{th}$ spare, $I_k(t)$ is defined by

$$(2.1) \qquad I_k(t) = A_k(t) - A_{k-1}(t) \qquad k = 1,......, n.$$

Before proceeding to a derivation of expressions for $I_n(t)$ and $A_n(t)$ in a general case, two special cases are considered; repair rate $\eta$ equal to infinity, and repair rate $\eta$ equal to failure rate $\lambda$. These are boundary cases for the cases of likely practical interest, in which it is reasonable to expect that repair rate will exceed failure rate.

In any case, $A_0(t)$, availability at time t with no spares is given by

(2.2) $$A_0(t) = P[T_0 > t] = e^{-\lambda t}, \quad t \geq 0.$$

In the following sections, it will be convenient to let

(2.3) $$U_n = T_0 + \ldots\ldots + T_n,$$

$$V_n = S_0 + \ldots\ldots + S_n,$$

$$W_n = U_n + V_n$$

$$= (S_0 + T_0) + \ldots\ldots + (S_n + T_n).$$

A. REPAIR RATE EQUAL TO INFINITY

The simplest case is the one in which no time is required to repair a failed unit, provided a spare unit is available.

In this case the contribution of the first spare is

(2.4) $$I_1(t) = \int_0^t P[T_1 > t-s \mid U_0 = s] \, f_{U_o}(s)ds$$

where

$$f_{U_0}(s) = \lambda e^{-\lambda s}, \quad s \geq 0 ,$$

is the gamma $\{1,\lambda\}$ density. Thus

(2.5) $$I_1(t) = \int_0^t e^{-\lambda(t-s)} \lambda e^{-\lambda s} \, ds$$

$$= \lambda e^{-\lambda s} \quad t,$$

and the availability of a system having one spare is

$$A_1(t) = A_0(t) + I_1(t) \tag{2.6}$$

$$= e^{-\lambda t}(1 + \lambda t)$$

The contribution of the second spare is

$$I_2(t) = \int_0^t P[T_2 > t-s \mid U_1 = s] f_{U_1}(s)\, ds \ , \tag{2.7}$$

where

$$f_{U_1}(s) = \frac{\lambda^2 s e^{-\lambda s}}{\Gamma(2)} \ , \ s \geq 0 \ ,$$

is the gamma $\{2,\lambda\}$ density. Thus

$$I_2(t) = \int_0^t e^{-\lambda(t-s)} \frac{\lambda^2 s e^{-\lambda s}}{\Gamma(2)}\, ds \tag{2.8}$$

$$= (\lambda t)^2 e^{-\lambda t} \frac{1}{2!} \ ,$$

and the availability of a system having two spares is

$$A_2(t) = e^{-\lambda t}\left(1 + \lambda t + \frac{(\lambda t)^2}{2!}\right) . \tag{2.9}$$

Generally, the contribution of the $n^{th}$ spare is

$$(2.10) \qquad I_n(t) = \int_0^t P[T_n > t-s \mid U_{n-1} = s] f_{U_{n-1}}(s)\, ds ,$$

where

$$f_{U_{n-1}}(s) = \frac{\lambda^n s^{n-1} e^{-\lambda s}}{\Gamma(n)} , \quad s \geq 0 ,$$

is the gamma $\{n,\lambda\}$ density. Thus

$$(2.11) \qquad I_n(t) = \int_0^t e^{-\ (t-s)} \frac{\lambda^n s^{n-1} e^{-\lambda s}}{\Gamma(n)}\, ds$$

$$= (\lambda t)^n e^{-\lambda t} \frac{1}{n!} ,$$

and the availability of a system having n spares is

$$(2.12) \qquad A_n(t) = e^{-\lambda t}\left(1 + \lambda t + \frac{(\lambda t)^2}{2!} + \ldots\ldots + \frac{(\lambda t)^n}{n!}\right) .$$

Availabilities $A_n(t)$ obtained from equation (2.12) are shown in table 1. In this table n represents the number of spares, and the contribution $I_n(t)$ of the $n^{th}$ spare can be found by subtracting $A_{n-1}(t)$ from $A_n(t)$. The availabilities shown in the last column are for an alternating renewal process with an infinite number of failure-repair cycles. This corresponds to having an infinite number of spares.

## B. REPAIR RATE EQUAL TO FAILURE RATE

The failure-repair process considered is that in which failure times and repair times are exponentially distributed with equal rates, i.e., $\lambda = \eta$.

Table 1 $A_n(t)$ for the case $\eta t = \infty$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9735 | .9978 | .9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| .50 | .9098 | .9856 | .9982 | .9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| .75 | .8266 | .9595 | .9927 | .9989 | .9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 1 | .7358 | .9197 | .9810 | .9963 | .9994 | .9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 2 | .4060 | .6767 | .8571 | .9473 | .9834 | .9955 | .9989 | .9998 | 1.0000 | 1.0000 | 1.0000 |
| 3 | .1991 | .4232 | .6472 | .8153 | .9161 | .9665 | .9881 | .9962 | .9989 | .9997 | 1.0000 |
| 4 | .0916 | .2381 | .4335 | .6288 | .7851 | .8893 | .9489 | .9786 | .9919 | .9972 | 1.0000 |
| 5 | .0404 | .1247 | .2650 | .4405 | .6160 | .7622 | .8666 | .9319 | .9682 | .9863 | 1.0000 |
| 6 | .0174 | .0620 | .1512 | .2851 | .4457 | .6063 | .7440 | .8472 | .9161 | .9574 | 1.0000 |
| 7 | .0073 | .0296 | .0818 | .1730 | .3007 | .4497 | .5987 | .7291 | .8305 | .9015 | 1.0000 |

In this case, the contribution of the first spare is

$$(2.13) \qquad I_1(t) = \int_0^t P[T_1 > t-s \mid W_0 = s] f_{W_0}(s)\, ds \ ,$$

where

$$f_{W_0}(s) = \frac{\lambda^2 s e^{-\lambda s}}{\Gamma(2)} \ , \ s \geq 0 \ ,$$

is the gamma $\{2,\lambda\}$ density. Thus

$$(2.14) \qquad I_1(t) = \int_0^t e^{-\lambda(t-s)} \frac{\lambda^2 s e^{-\lambda s}}{\Gamma(2)}\, ds$$

$$= \frac{\lambda^2 e^{-\lambda t}}{\Gamma(2)} \frac{t^2}{2} \ ,$$

and the availability of a system having one spare is

$$A_1(t) = A_0(t) + I_1(t)$$

$$= e^{-\lambda t} \left(1 + \frac{(\lambda t)^2}{2!}\right) \ .$$

Generally, the contribution of the $n^{th}$ spare is

$$(2.15) \qquad I_n(t) = \int_0^t P[T_n > t-s \mid W_{n-1} = s] f_{W_{n-1}}(s)\, ds \ ,$$

where

$$f_{W_{n-1}}(s) = \frac{\lambda^{2n} s^{2n-1} e^{-\lambda s}}{\Gamma(2n)} \ , \ s \geq 0 \ ,$$

is the gamma $\{2n,\lambda\}$ density. Thus

$$(2.16) \qquad I_n(t) = \int_0^t e^{-\lambda(t-s)} \frac{\lambda e^{-\lambda s}}{\Gamma(2n)} (\lambda s)^{2n-1} \, ds$$

$$= \frac{\lambda^{2n} e^{-\lambda t}}{(2n-1)!} \int_0^t s^{2n-1} \, ds$$

$$= \frac{(\lambda t)^{2n} e^{-\lambda t}}{(2n)!} ,$$

and the availability of a system having n spares is

$$(2.17) \qquad A_n(t) = e^{-\lambda t} \left(1 + \frac{(\lambda t)^2}{2!} + \cdots\cdots + \frac{(\lambda t)^{2n}}{(2n)!}\right) .$$

C. REPAIR RATE GREATER THAN FAILURE RATE

The failure-repair process considered in this section is visualized as one in which the repair rate is greater than the failure rate. This influences the format in which the results are displayed.

In this case, the contribution of the $n^{th}$ spare is

$$(2.18) \quad I_n(t) = \int_0^t P[T_n > t-s \mid W_{n-1} = s] \, f_{W_{n-1}}(s) \, ds ,$$

where

$$f_{W_{n-1}}(s) = \int_0^t f_{V_{n-1}}(s) \, f_{U_{n-1}}(t-s) \, ds ,$$

Table 2 $A_n(t)$ for the case $\eta t = \lambda t$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .8031 | .8033 | .0833 | .8033 | .8033 | .8033 | .8033 | .8033 | .8033 | .8033 | .8033 |
| .50 | .6823 | .6839 | .6839 | .6839 | .6839 | .6839 | .6839 | .6839 | .6839 | .6839 | .6839 |
| .75 | .6052 | .6114 | .6116 | .6116 | .6116 | .6116 | .6116 | .6116 | .6116 | .6116 | .6116 |
| 1 | .5518 | .5671 | .5677 | .5677 | .5677 | .5677 | .5677 | .5677 | .5677 | .5677 | .5677 |
| 2 | .4060 | .4962 | .5083 | .5091 | .5092 | .5092 | .5092 | .5092 | .5092 | .5092 | .5092 |
| 3 | .2738 | .4419 | .4923 | .5004 | .5012 | .5012 | .5012 | .5012 | .5012 | .5012 | .5012 |
| 4 | .1648 | .3602 | .4644 | .4942 | .4995 | .5001 | .5002 | .5002 | .5002 | .5002 | .5002 |
| 5 | .0910 | .2664 | .4127 | .4779 | .4961 | .4995 | .5000 | .5000 | .5000 | .5000 | .5000 |
| 6 | .0471 | .1809 | .3416 | .4448 | .4861 | .4974 | .4996 | .5000 | .5000 | .5000 | .5000 |
| 7 | .0233 | .1145 | .2635 | .3939 | .4648 | .4912 | .4983 | .4997 | .5000 | .5000 | .5000 |

and

$$f_{V_{n-1}}(s) = \frac{\eta^n}{\Gamma(n)} s^{n-1} e^{-\eta s} \quad , \quad s \geq 0 \; ,$$

is the gamma $\{n,\eta\}$ density,

while

$$f_{U_{n-1}}(t-s) = \frac{\lambda^n}{\Gamma(n)} (t-s)^{n-1} e^{-\lambda(t-s)} \quad , \quad t-s \geq 0 \; ,$$

is the gamma $\{n,\lambda\}$ density. Thus

$$(2.19) \quad I_n(t) = \int_0^t e^{-\lambda(t-s)} \int_0^s \frac{\eta^n}{\Gamma(n)} u^{n-1} e^{-\eta u} \frac{\lambda^n}{\Gamma(n)} (s-u)^{n-1} \cdot e^{-\lambda(t-s)} \, du \, ds \; .$$

Inverting the order of integration, equation (2.19) becomes

$$(2.20) \quad I_n(t) = \int_0^t \int_u^t e^{-\lambda(t-s)} \frac{\eta^n}{\Gamma(n)} u^{n-1} e^{-\eta u} \frac{\lambda^n}{\Gamma(n)} (s-u)^{n-1} \cdot e^{-\lambda(s-u)} \, ds \, du$$

$$= e^{-\lambda t} \frac{\eta^n \lambda^n}{\{\Gamma(n)\}^2} \int_0^t u^{n-1} e^{-(\eta-\lambda)u} \int_u^t (s-u)^{n-1} \, ds \, du \; .$$

Let $v = s - u$. Then

$$\int_0^t (s-u)^{n-1} \, ds = \int_0^{t-u} v^{n-1} \, dv = \frac{(t-u)^n}{n} \; .$$

Thus equation (2.20) reduces to

$$(2.21)\quad I_n(t) = e^{-\lambda t}\frac{\eta^n\lambda^n}{\Gamma(n)\Gamma(n+1)}\int_0^t u^{n-1}(t-u)^n e^{-(\eta-\lambda)u}\,du\ ,$$

and the availability of a system having n spares is

$$(2.22)\qquad A_n(t) = A_0(t) + I_1(t) + \ldots\ldots + I_n(t)\ .$$

Note than when $\eta = \lambda$, equation (2.21) reduces to equation (2.16), since then

$$I_n(t) = e^{-\lambda t}\ \frac{\lambda^{2n}}{\Gamma(n)\Gamma(n+1)}\int_0^t u^{n-1}(t-u)^n\ du\ .$$

Let $u = tv$. Then

$$\int_0^t u^{n-1}(t-u)^n\ du$$

$$= \int_0^t (tv)^{n-1}\{t(1-v)\}^n\ tdv$$

$$= t^{2n}\int_0^t v^{n-1}(1-v)^n\ dv$$

$$= t^{2n}\ \frac{\Gamma(n)\Gamma(n+1)}{\Gamma(2n+1)}$$

so that

$$I_n(t) = \frac{e^{-\lambda t}\lambda^{2n}}{\Gamma(n)\Gamma(n+1)}\, t^{2n}\, \frac{\Gamma(n)\Gamma(n+1)}{\Gamma(2n+1)}$$

$$= \frac{(\lambda t)^{2n} e^{-\lambda t}}{(2n)!} \quad .$$

## III. APPROXIMATION OF THE MATHEMATICAL MODEL

In section II, we derived a mathematical model for availability.

In this section, we discuss methods of approximation to obtain numerical values of availability.

### A. EXPONENTIAL EXPANSION APPROXIMATION

The integral in equation (2.21) can be approximated by expanding its exponential term, i.e.,

$$e^{-(\eta-\lambda)} = 1 - (\eta-\lambda)u + \frac{(\eta-\lambda)^2}{2!}u^2 - \ldots\ldots \tag{3.1}$$

Thus the integral becomes

$$\int_0^t u^{n-1}(t-u)^n \; \{1-(\eta-\lambda)u+\frac{(\eta-\lambda)^2}{2!}u^2 - \ldots\ldots\}\,du \tag{3.2}$$

$$= \int_0^t u^{n-1}(t-u)^n\,du$$

$$- (\eta-\lambda)\int_0^t u^n(t-u)^n\,du$$

$$+ \frac{(\eta-\lambda)^2}{2!}\int_0^t u^{n+1}(t-u)^n\,du$$

$$- \ldots\ldots\ldots\ldots$$

We know that,

$$\int_0^t u^{n-1}(t-u)^n \, du = t^{2n} \frac{\Gamma(n)\Gamma(n+1)}{\Gamma(2n+1)} \quad .$$

Thus equation (3.2) becomes

$$\text{(3.3)} \qquad I_n(t) = \frac{e^{-\lambda t}\eta^n\lambda^n}{\Gamma(n)\Gamma(n+1)} \{ t^{2n} \frac{\Gamma(n)\Gamma(n+1)}{\Gamma(2n+1)}$$

$$- (\eta-\lambda) t^{2n+1} \frac{\Gamma(n+1)\Gamma(n+1)}{\Gamma(2n+2)}$$

$$+ \frac{(\eta-\lambda)^2 t^{2n+2}}{2!} \frac{\Gamma(n+2)\Gamma(n+1)}{\Gamma(2n+3)}$$

$$- \ldots\ldots\ldots \}$$

$$= \frac{e^{-\lambda t}(\eta t)^n(\lambda t)^n}{\Gamma(n)} \{ \frac{\Gamma(n)}{\Gamma(2n+1)} - (\eta t-\lambda t) \frac{\Gamma(n+1)}{\Gamma(2n+2)}$$

$$+ \frac{(\eta t-\lambda t)^2}{2!} \frac{\Gamma(n+2)}{\Gamma(2n+3)}$$

$$- \ldots\ldots \}$$

Computational experiments, with the approximation represented by equation (3.3) have indicated unsatisfactory convergence behavior when $(\eta t-\lambda t)$ is large, a case of some practical interest, and so this approach was not pursued.

## B. SIMPSON'S RULE APPROXIMATION

The integral in equation (2.21) can be approximated by using Simpson's rule.

Let $v = \frac{u}{t}$. Then

$$(3.4) \qquad \int_0^t u^{n-1}(t-u)e^{-(\eta-\lambda)u}\,du$$

$$= \int_0^1 (tv)^{n-1}\{t(1-v)\}^n e^{-(\eta-\lambda)tv} t\,dv$$

$$= t^{2n}\int_0^1 v^{n-1}(1-v)^n e^{-(\eta t-\lambda t)v}\,dv .$$

Thus equation (2.21) becomes

$$(3.5) \qquad I_n(t) = e^{-\lambda t}\frac{(\eta t)^n(\lambda t)^n}{\Gamma(n)\Gamma(n+1)}\int_0^1 v^n(1-v)^n e^{-(\eta t-\lambda t)v}\,dv .$$

Now Simpson's rule can be applied to the integral in equation (3.5) to obtain numerical values of availability.

Simpson's rule as applied is

$$(3.6) \int_a^b f(x)\,dx = \frac{h}{3}(y_1+4y_2+2y_3+4y_4+\dots+2y_{m-1}+4y_m+y_{m+1})$$

where

$$h = \frac{b-a}{m} \quad ,$$

and $\qquad X_1 = a,\ X_2 = a+h,\ \dots\dots,\ X_{m+1} = a+mh = b,$

while $y_1 = f(X_1)$, $y_2 = f(X_2)$, ......, $y_{m+1} = f(X_{m+1})$ .

# IV. TABLE

In this section experimental tables of availabilities $A_n(t)$ are shown, which were obtained from the mathematical model evaluated using Simpson's rule, i.e., using equation (3.6) to evaluate equation (3.5) with $m = 500$, $X_1 = 0.0001$ and $X_{501} = 0.9999$. The reason for choosing $m = 500$ is that computational experiments with choices of m greater than or equal to 500 gave a stable and accurate result.

In these tables n represents the number of spares, and the contribution of the $n^{th}$ spare $I_n(t)$ can be found by subtracting $A_{n-1}(t)$ from $A_n(t)$.

The availabilities shown in this section, Table 3-12, are for cases in which $\eta t > \lambda t$.

The availabilities shown in the last column are for an alternating renewal process with an infinite number of failure-repair cycles. This corresponds to having an infinite number of spares.

Table 3 $A_n(t)$ for $\eta t = 20$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(5)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9660 | .9863 | .9876 | .9876 | .9876 | .9876 | .9876 | .9876 | .9876 | .9876 | .9877 |
| .5 | .9016 | .9663 | .9748 | .9756 | .9756 | .9756 | .9756 | .9756 | .9756 | .9756 | .9756 |
| .75 | .8213 | .9372 | .9604 | .9635 | .9638 | .9638 | .9638 | .9638 | .9638 | .9638 | .9639 |
| 1. | .7347 | .8990 | .9432 | .9512 | .9523 | .9524 | .9524 | .9524 | .9524 | .9524 | .9524 |
| 2. | .4194 | .6855 | .8342 | .8902 | .9054 | .9085 | .9090 | .9091 | .9091 | .9091 | .9091 |
| 3. | .2152 | .4587 | .6714 | .7958 | .8480 | .8646 | .8686 | .8694 | .8695 | .8696 | .8696 |
| 4. | .1042 | .2812 | .4963 | .6701 | .7704 | .8138 | .8284 | .8323 | .8332 | .8333 | .8333 |
| 5. | .0487 | .1625 | .3429 | .5320 | .6728 | .7510 | .7845 | .7960 | .7991 | .7998 | .8000 |
| 6. | .0222 | .0900 | .2250 | .4014 | .5640 | .6752 | .7337 | .7581 | .7663 | .7686 | .7692 |
| 7. | .0100 | .0485 | .1421 | .2904 | .4551 | .5900 | .6746 | .7165 | .7333 | .7388 | .7407 |

Table 4 $A_n(t)$ for $\eta t = 40$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(1)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9698 | .9920 | .9937 | .9938 | .9938 | .9938 | .9938 | .9938 | .9938 | .9938 | .9938 |
| .5 | .9058 | .9760 | .9864 | .9875 | .9876 | .9876 | .9876 | .9876 | .9876 | .9876 | .9876 |
| .75 | .8242 | .9486 | .9765 | .9810 | .9815 | .9816 | .9816 | .9816 | .9816 | .9816 | .9816 |
| 1. | .7355 | .9099 | .9623 | .9735 | .9753 | .9756 | .9756 | .9756 | .9756 | .9756 | .9756 |
| 2. | .4127 | .6823 | .8480 | .9204 | .9444 | .9507 | .9521 | .9523 | .9524 | .9524 | .9524 |
| 3. | .2069 | .4416 | .6629 | .8112 | .8866 | .9169 | .9268 | .9295 | .9301 | .9302 | .9302 |
| 4. | .0975 | .2591 | .4672 | .6573 | .7888 | .8606 | .8924 | .9042 | .9078 | .9088 | .9091 |
| 5. | .0441 | .1421 | .3036 | .4923 | .6589 | .7748 | .8404 | .8711 | .8832 | .8873 | .8889 |
| 6. | .0195 | .0743 | .1854 | .3448 | .5174 | .6645 | .7661 | .8242 | .8522 | .8637 | .8696 |
| 7. | .0084 | .0374 | .1079 | .2285 | .3843 | .5424 | .6722 | .7603 | .8107 | .8853 | .8511 |

Table 5 $A_n(t)$ for $\eta t = 60$

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9710 | .9939 | .9957 | .9958 | .9958 | .9958 | .9958 | .9958 | .9958 | .9958 | .9959 |
| .5 | .9071 | .9792 | .9903 | .9916 | .9917 | .9917 | .9917 | .9917 | .9917 | .9917 | .9917 |
| .75 | .8251 | .9523 | .9819 | .9869 | .9875 | .9876 | .9876 | .9876 | .9876 | .9876 | .9877 |
| 1. | .7356 | .9133 | .9686 | .9810 | .9832 | .9835 | .9836 | .9836 | .9836 | .9836 | .9836 |
| 2. | .4104 | .6807 | .8516 | .9298 | .9575 | .9654 | .9673 | .9677 | .9677 | .9677 | .9677 |
| 3. | .2042 | .4356 | .6584 | .8139 | .8977 | .9341 | .9471 | .9511 | .9521 | .9523 | .9524 |
| 4. | .0954 | .2519 | .4563 | .6494 | .7902 | .8728 | .9129 | .9293 | .9351 | .9369 | .9375 |
| 5. | .0428 | .1360 | .2905 | .4759 | .6475 | .7751 | .8537 | .8945 | .9127 | .9197 | .9231 |
| 6. | .0187 | .0698 | .1733 | .3247 | .4954 | .6500 | .7658 | .8389 | .8786 | .8973 | .9091 |
| 7. | .0080 | .0345 | .0983 | .2088 | .3565 | .5150 | .6554 | .7603 | .8275 | .8649 | .8955 |

Table 6 $A_n(t)$ for $\eta t = 80$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9716 | .9949 | .9967 | .9968 | .9968 | .9968 | .9968 | .9968 | .9968 | .9968 | .9969 |
| .5 | .9077 | .9808 | .9922 | .9936 | .9937 | .9937 | .9937 | .9937 | .9937 | .9937 | .9938 |
| .75 | .8253 | .9541 | .9845 | .9898 | .9905 | .9906 | .9906 | .9906 | .9906 | .9906 | .9907 |
| 1. | .7355 | .9149 | .9717 | .9848 | .9872 | .9875 | .9876 | .9876 | .9876 | .9876 | .9877 |
| 2. | .4093 | .6798 | .8531 | .9344 | .9641 | .9729 | .9750 | .9755 | .9756 | .9756 | .9756 |
| 3. | .2029 | .4325 | .6559 | .8148 | .9028 | .9425 | .9574 | .9622 | .9635 | .9638 | .9639 |
| 4. | .0944 | .2484 | .4507 | .6448 | .7899 | .8780 | .9226 | .9419 | .9491 | .9515 | .9524 |
| 5. | .0422 | .1330 | .2840 | .4673 | .6406 | .7736 | .8588 | .9052 | .9272 | .9363 | .9412 |
| 6. | .0183 | .0677 | .1675 | .3146 | .4835 | .6408 | .7630 | .8439 | .8903 | .9136 | .9302 |
| 7. | .0078 | .0332 | .0938 | .1994 | .3424 | .4997 | .6439 | .7565 | .8326 | .8776 | .9195 |

Table 7 $A_n(t)$ for $\eta t = 100$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9719 | .9954 | .9973 | .9974 | .9974 | .9974 | .9974 | .9974 | .9974 | .9974 | .9975 |
| .5 | .9081 | .9817 | .9934 | .9947 | .9949 | .9949 | .9949 | .9949 | .9949 | .9949 | .9950 |
| .75 | .8255 | .9551 | .9861 | .9916 | .9923 | .9924 | .9924 | .9924 | .9924 | .9924 | .9925 |
| 1. | .7355 | .9158 | .9735 | .9870 | .9895 | .9899 | .9900 | .9900 | .9900 | .9900 | .9901 |
| 2. | .4085 | .6792 | .8540 | .9371 | .9680 | .9774 | .9797 | .9803 | .9804 | .9804 | .9804 |
| 3. | .2020 | .4307 | .6543 | .8151 | .9058 | .9474 | .9636 | .9689 | .9704 | .9708 | .9709 |
| 4. | .0938 | .2464 | .4473 | .6419 | .7894 | .8808 | .9283 | .9494 | .9576 | .9604 | .9615 |
| 5. | .0418 | .1313 | .2802 | .4621 | .6362 | .7721 | .8613 | .9113 | .9358 | .9464 | .9524 |
| 6. | .0181 | .0665 | .1641 | .3086 | .4762 | .6347 | .7605 | .8461 | .8967 | .9232 | .9434 |
| 7. | .0077 | .0324 | .0913 | .1938 | .3340 | .4901 | .6361 | .7529 | .8343 | .8843 | .9346 |

Table 8 $A_n(t)$ for $\eta t = 120$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9721 | .9957 | .9976 | .9978 | .9978 | .9978 | .9978 | .9978 | .9978 | .9978 | .9979 |
| .5 | .9982 | .9822 | .9941 | .9955 | .9956 | .9956 | .9956 | .9956 | .9956 | .9956 | .9956 |
| .75 | .8255 | .9557 | .9871 | .9927 | .9935 | .9936 | .9936 | .9936 | .9936 | .9936 | .9938 |
| 1. | .7354 | .9164 | .9747 | .9885 | .9911 | .9915 | .9916 | .9916 | .9916 | .9916 | .9917 |
| 2. | .4080 | .6788 | .8546 | .9388 | .9706 | .9804 | .9829 | .9835 | .9836 | .9836 | .9836 |
| 3. | .2015 | .4295 | .6532 | .8153 | .9076 | .9507 | .9677 | .9734 | .9751 | .9756 | .9756 |
| 4. | .0934 | .2450 | .4451 | .6399 | .7890 | .8825 | .9320 | .9544 | .9633 | .9665 | .9677 |
| 5. | .0416 | .1302 | .2776 | .4585 | .6331 | .7709 | .8627 | .9152 | .9415 | .9531 | .9600 |
| 6. | .0180 | .0657 | .1619 | .3046 | .4712 | .6304 | .7585 | .8471 | .9007 | .9293 | .9524 |
| 7. | .0076 | .0320 | .0896 | .1902 | .3284 | .4836 | .6305 | .7500 | .8349 | .8883 | .9449 |

Table 9 $A_n(t)$ for $\eta t = 140$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9722 | .9960 | .9979 | .9980 | .9980 | .9980 | .9980 | .9980 | .9980 | .9980 | .9982 |
| .5 | .9083 | .9826 | .9946 | .9960 | .9961 | .9961 | .9961 | .9961 | .9961 | .9961 | .9964 |
| .75 | .8255 | .9562 | .9878 | .9935 | .9943 | .9944 | .9944 | .9944 | .9944 | .9944 | .9947 |
| 1. | .7353 | .9168 | .9755 | .9895 | .9922 | .9926 | .9927 | .9927 | .9927 | .9927 | .9929 |
| 2. | .4076 | .6785 | .8550 | .9401 | .9724 | .9825 | .9852 | .9858 | .9859 | .9859 | .9859 |
| 3. | .2011 | .4286 | .6524 | .8154 | .9090 | .9531 | .9706 | .9767 | .9785 | .9790 | .9790 |
| 4. | .0931 | .2440 | .4435 | .6384 | .7887 | .8837 | .9345 | .9580 | .9674 | .9708 | .9722 |
| 5. | .0414 | .1294 | .2758 | .4560 | .6308 | .7700 | .8636 | .9179 | .9455 | .9579 | .9655 |
| 6. | .0179 | .0652 | .1603 | .3018 | .4676 | .6272 | .7568 | .8477 | .9034 | .9337 | .9589 |
| 7. | .0076 | .0316 | .0884 | .1877 | .3244 | .4789 | .6263 | .7477 | .8350 | .8909 | .9524 |

Table 10 $A_n(t)$ for $\eta t = 160$

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9723 | .9961 | .9980 | .9982 | .9982 | .9982 | .9982 | .9982 | .9982 | .9982 | .9984 |
| .5 | .9083 | .9829 | .9949 | .9964 | .9965 | .9965 | .9965 | .9965 | .9965 | .9965 | .9969 |
| .75 | .8255 | .9565 | .9883 | .9940 | .9949 | .9949 | .9950 | .9950 | .9950 | .9950 | .9953 |
| 1. | .7351 | .9170 | .9761 | .9903 | .9930 | .9934 | .9935 | .9935 | .9935 | .9935 | .9938 |
| 2. | .4073 | .6783 | .8553 | .9410 | .9738 | .9841 | .9869 | .9875 | .9876 | .9876 | .9877 |
| 3. | .2008 | .4279 | .6519 | .8155 | .9100 | .9548 | .9729 | .9791 | .9810 | .9816 | .9816 |
| 4. | .0929 | .2433 | .4423 | .6373 | .7884 | .8846 | .9365 | .9606 | .9705 | .9741 | .9756 |
| 5. | .0412 | .1288 | .2745 | .4541 | .6291 | .7692 | .8642 | .9199 | .9484 | .9615 | .9697 |
| 6. | .0178 | .0648 | .1592 | .2997 | .4649 | .6248 | .7555 | .8480 | .9053 | .9369 | .9639 |
| 7. | .0075 | .0314 | .0876 | .1858 | .3214 | .4753 | .6231 | .7458 | .8350 | .8927 | .9581 |

Table 11 $A_n(t)$ for $\eta t$ = 180

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9723 | .9962 | .9981 | .9983 | .9983 | .9983 | .9983 | .9983 | .9983 | .9983 | .9986 |
| .5 | .9083 | .9830 | .9951 | .9966 | .9967 | .9967 | .9968 | .9968 | .9968 | .9968 | .9972 |
| .75 | .8254 | .9567 | .9887 | .9944 | .9953 | .9954 | .9954 | .9954 | .9954 | .9954 | .9959 |
| 1. | .7350 | .9172 | .9965 | .9908 | .9936 | .9940 | .9941 | .9941 | .9941 | .9941 | .9945 |
| 2. | .4070 | .6781 | .8555 | .9417 | .9749 | .9854 | .9882 | .9888 | .9890 | .9890 | .9890 |
| 3. | .2005 | .4274 | .6514 | .8156 | .9107 | .9562 | .9746 | .9811 | .9831 | .9836 | .9836 |
| 4. | .0927 | .2427 | .4414 | .6365 | .7881 | .8853 | .9380 | .9627 | .9729 | .9767 | .9783 |
| 5. | .0411 | .1284 | .2735 | .4527 | .6277 | .7686 | .8647 | .9214 | .9508 | .9643 | .9730 |
| 6. | .0177 | .0645 | .1583 | .2981 | .4629 | .6228 | .7545 | .8481 | .9067 | .9393 | .9677 |
| 7. | .0075 | .0312 | .0869 | .1844 | .3191 | .4725 | .6206 | .7442 | .8348 | .8940 | .9626 |

Table 12 $A_n(t)$ for $\eta t = 200$

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_n(t)$ | | | | | | | | | | | |
| $\lambda t$ | $A_1(t)$ | $A_2(t)$ | $A_3(t)$ | $A_4(t)$ | $A_5(t)$ | $A_6(t)$ | $A_7(t)$ | $A_8(t)$ | $A_9(t)$ | $A_{10}(t)$ | $A_\infty(t)$ |
| .25 | .9723 | .9962 | .9982 | .9983 | .9983 | .9983 | .9983 | .9983 | .9983 | .9983 | .9988 |
| .5 | .9083 | .9831 | .9953 | .9968 | .9969 | .9969 | .9969 | .9969 | .9969 | .9969 | .9975 |
| .75 | .8253 | .9568 | .9989 | .9949 | .9956 | .9957 | .9957 | .9957 | .9957 | .9957 | .9963 |
| 1. | .7348 | .9173 | .9768 | .9913 | .9940 | .9945 | .9945 | .9945 | .9945 | .9945 | .9950 |
| 2. | .4067 | .6779 | .8557 | .9423 | .9757 | .9864 | .9892 | .9899 | .9901 | .9901 | .9901 |
| 3. | .2003 | .4270 | .6511 | .8156 | .9113 | .9573 | .9760 | .9826 | .9847 | .9852 | .9852 |
| 4. | .0925 | .2423 | .4406 | .6358 | .7879 | .8858 | .9392 | .9644 | .9749 | .9788 | .9804 |
| 5. | .0410 | .1280 | .2727 | .4515 | .6266 | .7681 | .8650 | .9225 | .9526 | .9666 | .9756 |
| 6. | .0177 | .0642 | .1576 | .2968 | .4612 | .6213 | .7536 | .8482 | .9078 | .9413 | .9709 |
| 7. | .0075 | .0310 | .0864 | .1832 | .3172 | .4703 | .6185 | .7429 | .8347 | .8950 | .9662 |

## V. SUMMARY AND CONCLUSIONS

Certain computational approaches were tried for obtaining availabilities for a device supported by only a finite backlog of spares, using the simple assumptions that failure and repair rates are constant. Real failure and repair distributions may be more complex, but the case considered is a good case for initial computational experiments.

Of the two approaches tried, neither proved entirely satisfactory in obtaining availabilities in a way that is fast and suitable for use with small-scale computational facilities, e.g.. hand-held calculators. Also neither was effective over the entire range of failure rate and repair rate combinations that might be of interest.

Since an easily used, readily accessible,way to assess the impact of finite spares backlogs on availability is desirable in many mission planning contexts, further computational approaches should be tried.

The tables presented in section IV give availability values with which the result of such experiments can be compared.

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